Period 1, Feb 12, 2025

BASIC INTEGRATION RULES ($a > 0$) Pg. 385

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$
10. $\int \tan u\,du = -\ln|\cos u| + C$
11. $\int \cot u\,du = \ln|\sin u| + C$
12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$
16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

d) $\int \frac{1}{x\sqrt{4x^2 - 36}}\,dx$

$u^2 - 6^2$

$u = 2x \Rightarrow \frac{u}{2} = x$

$u^2 = 4x^2$

$du = 2dx \Rightarrow \frac{du}{2} = dx$

$\frac{du}{2} = dx$

$u = 2x \quad a = 6$

$$\int \frac{1}{\frac{u}{2}\sqrt{u^2-6^2}} \cdot \frac{du}{2}$$

$$\int \frac{du}{\frac{u}{\cancel{2}}\sqrt{u^2-6^2} \cdot \cancel{2}} = \int \frac{du}{u\sqrt{u^2-6^2}} = \frac{1}{6} \operatorname{arcsec} \frac{|2x|}{6} + C$$

b) $\int \frac{4}{25x^2+9}dx = \int \frac{4}{(5x)^2+3^2}dx$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$5x = u$

$5dx = du$

$dx = \frac{du}{5}$

$\int \frac{4}{u^2+3^2}\cdot\frac{du}{5} = \frac{4}{5}\int\frac{du}{u^2+3^2}$

$u = 5x \quad a = 3$

$\frac{4}{5}\cdot\frac{1}{3}\arctan\frac{5x}{3} + C$

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c) $\int \frac{e^x}{4+e^{2x}}dx$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$u = e^x$

$u^2 = e^x\cdot e^x = e^{2x}$

$du = e^x dx$

$\frac{du}{e^x} = dx$

$\int \frac{\cancel{e^x}}{2^2+u^2}\cdot\frac{du}{\cancel{e^x}}$

$\int \frac{du}{2^2+u^2}$ $\quad a = 2$, $u = e^x$

$\frac{1}{2}\cdot\arctan\frac{e^x}{2} + C$

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b) $\int \tan^2 x \sec^2 x\, dx$

$u = \tan x$

$du = \sec^2 x\, dx$

$\frac{du}{\sec^2 x} = dx$

$\int u^2 \cancel{\sec^2 x}\cdot\frac{du}{\cancel{\sec^2 x}}$

$\int u^2 du = \frac{1}{3}u^{2+1=3} + C$

$\frac{1}{3}u^3 + C = \frac{1}{3}(\tan x)^3 + C = \frac{\tan^3 x}{3} + C$

Evaluate each integral.

a) $\int (\tan x + 2\sec x)\,dx$

10. $\int \tan u\,du = -\ln|\cos u| + C$

12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

$\int \tan x\,dx + \int 2\sec x\,dx$

$-\ln|\cos x| + 2\ln|\sec x + \tan x| + C$

$-\ln|\cos x| + \ln(\sec x + \tan x)^2$

$\ln(\sec x + \tan x)^2 - \ln|\cos x|$

$\ln \dfrac{(\sec x + \tan x)^2}{|\cos x|}$

$\int 5x^2\,dx$

$5\int x^2\,dx$

$5 \cdot \frac{1}{3} x^{2+1=3} = \frac{5}{3}x^3 + C$

$$\int \frac{x-3}{\sqrt{9-x^2}}\,dx = \int \frac{x}{\sqrt{3^2-x^2}}\,dx - \int \frac{3}{\sqrt{9-x^2}}\,dx$$

$u = 9 - x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$\int \frac{\cancel{x}}{\sqrt{u}} \cdot \frac{du}{-2\cancel{x}}$

$-\frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$-\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{1} \cdot u^{-\frac{1}{2}+1=\frac{1}{2}} + C_1$

$-u^{\frac{1}{2}} + C$

$-\sqrt{9-x^2} + C_1$

18. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$

$3\int \frac{dx}{\sqrt{3^2-x^2}}$

$3\arcsin\frac{x}{3} + C$

$-\sqrt{9-x^2} - 3\arcsin\frac{x}{3} + C$

$$\int \frac{dx}{x^2+6x+25}$$

$$\int \frac{dx}{\underbrace{x^2+6x+9}\underbrace{-9+25}}$$

$$\int \frac{dx}{(x+3)^2+16}$$

$$\int \frac{dx}{(x+3)^2+4^2} = \int \frac{du}{u^2+4^2} \rightarrow$$

$$\text{19. } \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$u = x+3$
$du = dx$
$a = 4$

$$\frac{1}{4}\arctan\frac{x+3}{4} + C$$

ax^2+bx+c

$x^2+6x+9 = (x+3)^2$

$a = 1$
$b = 6$
$\frac{b}{2} = \frac{6}{2} = 3$
$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

$$\int \frac{x+7}{x^2+6x+25}\,dx = \int \frac{x+3}{x^2+6x+25}\,dx + \int \frac{4}{x^2+6x+25}\,dx$$

$$\int \frac{x+3+4}{x^2+6x+25}\,dx$$

$u = x^2+6x+25$
$du = (2x+6)\,dx$

$$\frac{du}{2x+6} = dx \Rightarrow \frac{du}{2(x+3)} = dx$$

$$\int \frac{\cancel{x+3}}{u} \cdot \frac{du}{2(\cancel{x+3})}$$

$$\frac{1}{2}\int \frac{du}{u}$$

$$\frac{1}{2}\ln|u|$$

$$\int \frac{4}{(x+3)^2+16}\,dx$$

$x+3 = u$
$4 = a$

$$4\int \frac{dx}{u^2+a^2}$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\cancel{4} \cdot \frac{1}{\cancel{4}}\arctan\frac{x+3}{4} + c$$

$$\frac{1}{2}\ln|x^2+6x+25| + \arctan\frac{x+3}{4} + C$$

$$\ln\left|\sqrt{x^2+6x+25}\right| + \arctan\frac{x+3}{4} + C$$

$$\int x\sqrt{2x-1}\,dx = \int x\sqrt{u}\cdot\frac{du}{2} = \int \frac{(u+1)}{2}\cdot u^{\frac12}\cdot\frac{du}{2} = \frac14\int (u+1)u^{\frac12}\,du$$

$u=2x-1 \Rightarrow u+1=2x \Rightarrow \frac{u+1}{2}=x$

$du=2dx$

$\frac{du}{2}=dx$

$$\frac14\int\left(u^{\frac32}+u^{\frac12}\right)du$$

$$\frac14\left[\frac25 u^{3/2+1=\frac52} + \frac23 u^{\frac12+1=\frac32}\right]+C$$

$$\frac14\left[\frac25(2x-1)^{5/2}+\frac23(2x-1)^{3/2}\right]+C$$

$$\frac14\sqrt{2x-1}\left[\frac25(2x-1)^2+\frac23(2x-1)^1\right]+C$$

$$\frac14\left[\frac25(2x-1)^2(2x-1)^{\frac12}+\frac23(2x-1)^1(2x-1)^{\frac12}\right]$$

$$\int\frac{2x}{(x+1)^2}\,dx = \int\frac{2x}{u^2}\cdot du = 2\int\frac{u-1}{u^2}\,du = 2\int\left[\frac{u}{u^2}-\frac{1}{u^2}\right]du$$

$u=x+1 \Rightarrow u-1=x$

$du=dx$

$$2\int\left[\frac1u - u^{-2}\right]du$$

$$2\left[\ln|u| - \frac{1}{-1}\cdot u^{-2+1=-1}\right]+C$$

$$2\ln|u| + 2u^{-1}+C$$

$$2\ln|x+1| + \frac{2}{x+1}+C$$

$$\int \frac{x^2+x+1}{x^2+1}dx$$

$$x^2+1 \overline{\big) x^2+x+1} \quad 1 + \frac{x}{x^2+1}$$

$$-(x^2+0x+1)$$

$$0 \quad +x \quad 0$$

$$\int \left(\frac{x^2+1}{x^2+1} + \frac{x}{x^2+1}\right) dx$$

$$\int \left(1 + \frac{x}{x^2+1}\right)dx = \int 1\,dx + \int \frac{x}{x^2+1}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$x \qquad \int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u|$$

$$= x + \frac{1}{2}\ln|x^2+1| + C$$

$$\int_0^2 x\sqrt{4-x^2}\ dx$$

$u = 4 - x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$x=2$, $u=0$

$x=0$, $u=4$

U-bounds

$$\int_4^0 -\frac{1}{2}u^{\frac{1}{2}}du$$

$$-\frac{1}{2}\int_4^0 u^{\frac{1}{2}}du$$

$$-\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} u^{3/2} + C$$

$$-\frac{1}{3}(0)^{3/2} - \left[-\frac{1}{3}(4)^{3/2}\right] = 0 + \frac{1}{3}\cdot 8 = \frac{8}{3}$$

X-bounds

$$\int x\sqrt{4-x^2}\ dx$$

$$\int \cancel{x}\sqrt{u} \cdot \frac{du}{-2\cancel{x}}$$

$$-\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$-\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} u^{\frac{1}{2}+1 = \frac{3}{2}} + C$$

$$-\frac{1}{3}u^{3/2} + C$$

$$-\frac{1}{3}(4-x^2)^{3/2} + C \Big[_0^2$$

$$-\frac{1}{3}(4-2^2)^{3/2} - \left[-\frac{1}{3}(4-0^2)^{3/2}\right]$$

$$0 + \frac{1}{3}\cdot 8 = \frac{8}{3}$$

$$\int_0^{\frac{\pi}{4}} \tan x \ \sec^2 x \ dx$$

$U = \tan x$

$dU = \sec^2 x \, dx$

$\frac{dU}{\sec^2 x} = dx$

$U = \tan x$

$x = 0 \quad U = \tan 0$

$U = 0$

$x = \frac{\pi}{4}$

$U = \tan\frac{\pi}{4}$

$U = 1$

$\int_0^1 U \cdot \cancel{\sec^2 x} \frac{dU}{\cancel{\sec^2 x}}$

$\int_0^1 U \, dU = \frac{1}{2}U^2 + C \Big]_0^1$

$\frac{1}{2}(1)^2 - \frac{1}{2}(0)^2 = \frac{1}{2}$

$$\int_{-1}^{1} 3x^2 \sqrt{x^3+1} \, dx$$

$U = x^3 + 1$

$dU = 3x^2 dx$

$\frac{dU}{3x^2} = dx$

$x = -1$

$U = (-1)^3 + 1$

$U = -1 + 1$

$x = -1, U = 0$

$U = x^3 + 1$

$x = 1$

$U = (1)^3 + 1$

$U = 1 + 1$

$U = 2$

$\int_0^2 \cancel{3x^2} \sqrt{U} \cdot \frac{dU}{\cancel{3x^2}}$

$\int_0^2 U^{\frac{1}{2}} dU = \frac{2}{3} \cdot U^{\frac{1}{2}+1=\frac{3}{2}} + C$

$\frac{2}{3} U^{3/2} + C \Big]_0^2$

$\frac{2}{3}(2)^{3/2} - \frac{2}{3}(0)^{3/2} = \frac{2 \cdot 2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$

$$\int \frac{dx}{\sqrt{x^2 - 1}} \qquad (Can't\ do.)$$

$\int \frac{dx}{\sqrt{-1(1-x^2)}} = \int \frac{dx}{i\sqrt{1-x^2}}$

18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

$\frac{dU}{\sqrt{1-x^2}}$